Harsh Bohra of Baker Consulting Group proposed the following modifications to PEMY's proposal for API 653, Ballot 6014 - API 653-1012 Overhaul of Differential Settlement Methods:

1. **Retention of Existing Marr Method**: *"We propose that the existing Marr method be retained as an alternative approach within the settlement guidelines"*.
2. **Minimum Fourier Terms**: *"We suggest that the requirement for using a minimum of six Fourier terms be added for calculating the second derivative. While this primarily impacts tanks with smaller diameters, we believe it's crucial to ensure accurate data capture and decision-making"*.

Commentary on Marr Method usage

The Marr method can be retained as an alternative approach to Andreani but should have restrictions on its usage.

In a white paper[1] we reported that both methods (Marr and Andreani) that are sanctioned by the current API 653 are based on three-point estimates of the curvature (second derivative) of the out-of-plane deflection curve. When we re-expressed Marr's and Andreani's criteria as upper bounds on the second derivative, we found that Marr's bound was much higher than Andreani's for inter-point spacing larger than 8 feet (the minimum allowed by API 653). And we were able to explain this: Marr's three-point (second divided difference) curvature estimate grossly underestimates the exact second derivative by at least a factor of 2 when the spacing (L) between measurements exceeds 20 feet (Figure 1).

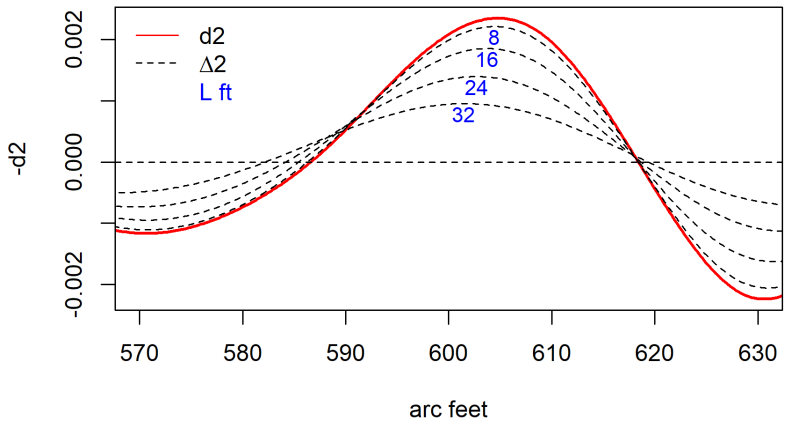

Figure 1. Marr's second difference underestimates second derivative.

Andreani's upper bound on estimated curvature avoids this bias and has been calibrated with a series of over 3000 finite element analyses of single folds along chords across a tank bottom with arc lengths 20, 30, 40, …, C/4, C/2, where C is the circumference of the tank; in these FEA's, tank diameters were D = 50, 80, 120, 180, 240, 300. However; it has not been calibrated for arc lengths shorter than 20 or for more complex out-of-plane settlements.

We also reported that controlling Type II errors (false negatives: undetected overstress) requires at least a 5-sigma signal to noise ratio (defined as maximum permitted curvature divided by residual standard error) and that is not attainable for manual elevation measurements closer than 16 feet.

Consequently, we find that, for inter-point distances between 16 and 20 feet, Marr's method applied to manual measurements is both relatively unbiased, and relatively insensitive to noise. Table 1 shows combinations of tank diameter and number of manual measurement stations for which Marr's method is permitted, in some cases (highlighted in blue) by using every other measurement. For uncoloured cells, Marr's method is strongly discouraged on account of bias or noise sensitivity. In those situations, Andreani's method is preferred instead.

[1] See *Commentary on API 653 Annex B "Evaluation of Tank Bottom Settlement"*, from Ballot 6014 - API 653-1012 Overhaul of Differential Settlement Methods

Diam	Circum	Number of manual measurement stations																										
		6	8	10	12	14	16	18	20	22	24	26	28	32	36	40	42	44	46	48	50	52	54	56	58	60	62	
32	100	17			17												XX											Marr: Distance between points, ft
48	150		19				19	17									XX											Marr/skip 1: Distance between points, ft
64	200			20	17				20	18	17																	
80	250				21	18	16				21	19	18	16														
95	300					21	19	17					21	19	17													
111	350						22	19	18	16				22	19	18	17	16										
127	400								20	18	17					20	19	18	17	17								
143	450									20	19	17	16					20	20	19	18	17	17	16				
159	500										21	19	18	16							20	19	19	18	17	17	16	
175	550											21	20	17	15								20	20	19	18	18	
191	600												21	19	17											20	19	
207	650													20	18	16												
223	700													22	19	18	17	16										
239	750														21	19	18	17	16									
255	800															20	19	18	17	17	16							
271	850																20	19	18	18	17	16						
286	900																	20	20	19	18	17	17	16				
302	950																		21	20	19	18	18	17	16	16		
318	1000																				20	19	19	18	17	17	16	

Table 1. Suggested Permitted Applications of Marr's Method

Commentary on minimum Fourier series terms for the "Trig Reg" method

Marr's method, with optimal spacing between measurement stations of 16 to 20-ft, is too wasteful of data to be used for fine-grained measurement methods such as laser scanning. Our trigonometric (Fourier) regression method ("Trig Reg") was designed for fine-grained data. Although it is, in theory, capable of detecting periodicity up to the Nyquist frequency n/2, we have limited it to wavelengths of 32 feet or longer. The reason for that is to align Trig Reg with Marr and Andreani. Marr, at the shortest permitted point spacing, 8-ft, can detect a half wave of 16-ft (one positive- or negative-going deflection arc). Andreani is not limited by point-spacing but is limited by the fact that it is calibrated on simulated 20-foot or longer arcs.

For these reasons we enforce a minimum 32-foot wavelength on the Trig-Reg method. Consequently, the highest permitted frequency is Circumference/32 = $\pi D/32$. The software we have written begins by forcing in all frequencies up to the highest permitted and eliminating statistically insignificant frequencies. If C/32 >= 6, then the Trig-Reg method will force in at least 6 cycles; this happens for circumference greater than 6x32=192 feet or diameter greater than 61 feet.

For diameters less than 61 feet the software we have written advises to user to run Marr if permitted by Table 2 or to run Andreani.